Grade 8 Summer Learning

For students who have just completed Grade 8

Because work with proportional thinking is so important in Grades 8 and 9, it is logical to focus on percent and some linear relations in this review. As well, the multiplication and division of both integers and fractions are very important aspects of the Grade 8 math curriculum. The Pythagorean theorem is introduced in Grade 8 and built on in Grade 9, so this, too, is an important topic to review. And because algebra becomes increasingly important in going up the grades, it also makes sense to focus some summer learning on algebraic ideas, including the use of scatter plots.

Therefore, the Grade 8 topics I decided to focus on to ready students for Grade 9 are the following:

- Calculations involving percents
- Multiplication and division of integers
- Multiplication and division of fractions
- Evaluating algebraic expressions and solving equations
- The Pythagorean theorem
- Graphing linear patterns
- Scatter plots

Essential Understandings that are the focus of the support:

DE–4 A place-value system makes it easier to describe and compare numbers.

DE–6 Benchmark numbers can be used to compare and give meaning to decimal numbers.

RN–1 Every real number can be represented in many ways. Each way highlights something different about that real number.

R–1 When a rational number is used to describe a part of a whole, the whole must be known to make sense of the number.

O–3 Multiplication is about a change from a unit of a given size to a unit of 1. You know the size and the number of units (the size and the number of groups) and you multiply to figure out the number of units of 1 (the product).

O–4 Division is about a change from a unit of 1 to a unit of a given size. You know the number of units of 1 and the size of the unit (the dividend and the size of the group) and you divide to figure out the number of units (the number of groups). Or you know the number of units of 1 and the number of units (the dividend and the number of groups) and you divide to figure out the size of the unit (the size of the group).

O–5 There are relationships among the four operations. Addition and subtraction are inverse operations.

O–9 Estimating is an essential part of any computation to catch errors or to give a feel for how to proceed with a calculation.

O–10 There are always multiple strategies for determining the result of a computation, whether it is an estimated or an exact result.

O–14 Taking a square root can be thought of as the opposite of squaring.

PR–1 Sometimes it is useful to compare two numbers in terms of how far apart they are, but other times it is useful to compare them in terms of how many units of one number it would take to fit into the other.

PR–2 Any comparison involving a ratio can be thought of as a fraction and vice versa.

PR–5 Any rate or ratio relationship can be represented in different ways. Different representations might be useful in different situations.

A–1 Many of the properties that underlie operations are useful in certain circumstances to simplify calculations or to predict how specific values of expressions will change with a change in the value of a variable.

A–2 Equality is an expression of balance. The two sides of an equation describe the same quantity.

A–4 Often more than one equation or one set of related equations can describe a situation algebraically.

A–5 The use of algebraic notation often simplifies the description of a numerical relationship.

A–6 Solving an equation uses relationships between numbers and relationships between operations to determine an equivalent, simpler form of the equation.

DA–3 Often a visual data display makes it easier to show data. The type of graph used depends on what we want viewers to see, including frequency (how often something occurs), comparisons between categories, changes over time, and so on.

SO–2 Different tests can often be used to determine if an item is a certain kind of shape or object; many of these tests require measuring.

SO–4 Composing and decomposing a shape or object can provide information about the shape or object.

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This has been set up for 20 sessions of about 1.5 to 2 hours each:

- Each day includes at least one Number Talk.
- Each day also includes either a Diagnostic Task, which may be followed up with an additional Number Talk or some practice activities, or a MathUP lesson, which is followed up with practice activities.

Number Talks that are particularly recommended are the following:

Grade 8: 9, 10, 11, 12, 16, 17, 24, 25, 26, 36, 37, 39, 43, 47, 48, 52, 55, 56, 72, 75

Grade 9 Diagnostic Tasks to check on prerequisites from Grade 8 are provided as a PDF.

On a day that a Diagnostic Task is used (based on the seven focus topics), there is a Number Talk followed by the Diagnostic Task. The task should be described as an activity, not a test, to reduce any anxiety students might feel.

It might be appropriate to review some of the vocabulary in the Diagnostic Task before administering it.

If students struggle with the Diagnostic Task, it might be a good idea to go back to the related Grade 8 Diagnostic Tasks and treat them as additional activities. These tasks come from the following topics:

- Percent
- Integer Operations
- Fraction Operations
- Using Algebra
- Solving Equations and Inequalitites
- Displaying Data
- The Pythagorean Theorem

If there are no problems with the Diagnostic Task and you have more time to work with students, you might choose to work on additional Number Talks, or you might choose to use one or more of these Minds On activities from the following topics:

- Whole Number and Decimal Operations
- Large and Small Numbers
- Rational and Irrational Numbers
- Linear Patterns and Relationships
- Measurement
- The Pythagorean Theorem

The suggested MathUP lessons that follow assume that students are working at the Grade 8 level and that it is not necessary to return to lessons from an earlier grade.

Before beginning a lesson, it would be valuable for the teacher to read the Sum It UP section to review the content being covered and then move on to the three parts of the lesson — Minds On, Action, and Consolidate — followed by the Your Turn Questions and additional suggested practice activities.

Grade 8 Summer Learning

For students who have just completed Grade 8

Strand	Topic	Lesson	* Prerequisite Topics
N	Percent	**Lesson 1** Decimal Percents **Lesson 2** Percents Greater Than 100% **Lesson 4** Solving Problems When the Percent Is Known	None
N	Integer Operations	**Lesson 2** Representing the Multiplication of Integers **Lesson 4** Representing the Division of Integers	None
N	Fraction Operations	**Lesson 1** Comparing Fractions Using Multiplication **Lesson 2** Dividing to Determine a Unit Rate	None
A	Using Algebra *	**Lesson 1** Evaluating Expressions	Integer Operations
A	Solving Equations and Inequalities *	**Lesson 1** Solving Equations Using Informal Methods **Lesson 3** Solving Equations Algebraically by Balancing	Integer Operations Using Algebra
D	Displaying Data	**Lesson 1** Using Scatter Plots to Display Data	None
SS	The Pythagorean Theorem	**Lesson 1** Developing the Pythagorean Theorem Geometrically **Lesson 2** Applying the Pythagorean Theorem	Similarity Solving Equations and Inequalities

MATH UP Name: ______________________ Date: ________________

Percent

1. Fill in the two blanks with two numbers to make the sentence true.
   ___ is 75% of ____.

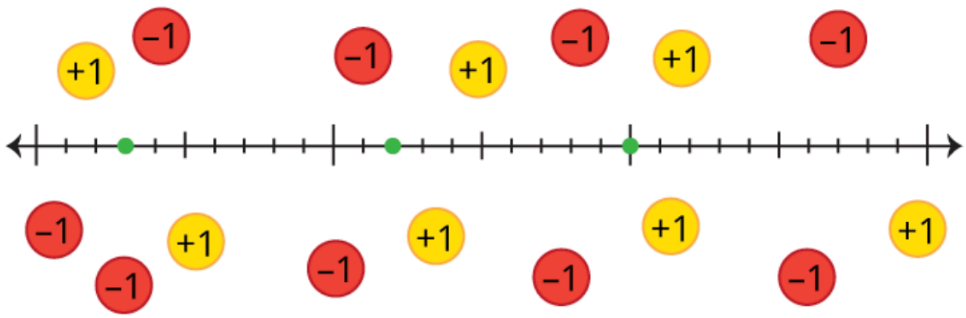
   Repeat using three other pairs of numbers.
   E.g., 75 and 100; 150 and 200; 45 and 60; 37.5 and 50

2. The sale price of an item is \$46 when the price is 30% off. About how much was the original price? Show your thinking.
   E.g., About \$70
   I figured out that \$46 is close to $\frac{2}{3}$ since 30% off is almost $\frac{1}{3}$ off. If $\frac{2}{3}$ is \$46, then $\frac{1}{3}$ is \$23, and so the whole price, which is $\frac{3}{3}$, is about \$69, so I said \$70 for an estimate.

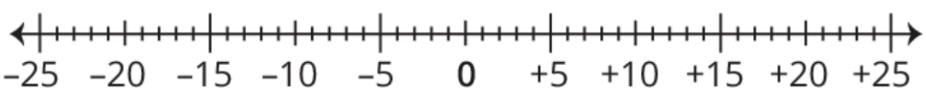
3. A price was reduced from \$65 to \$45. About how much was the percent discount? Show your thinking.
   E.g., About 30%
   The discount was \$20, and if you compare \$20 to \$65, it's about 13, so I said 30% as an estimate.

4. Estimate the sale price of an \$85.50 item if the discount is 15%.
   E.g., \$72
   Since 10% is about \$8.50 and half of that is \$4.25, 15% would be about \$13. So, I would estimate the sale price as about \$72.

Integers

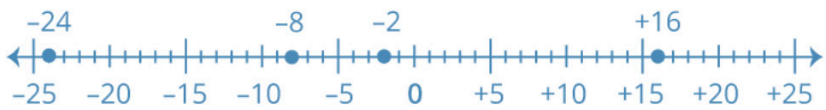


1. Show where each number would go on this number line.



−8 +16 −24 −2

E.g.,



2. Represent two of the numbers in Question 1 with integer counters. Sketch your representations.

E.g.,

Integers (continued)

3. What could −4 mean in each of these situations?

a) relative to sea level
E.g., 4 km below sea level

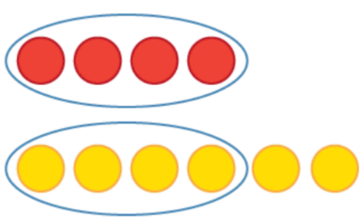
b) relative to your current location
E.g., 4 km to the west or 4 km south

c) using a thermometer
E.g., 4 degrees below 0°

d) in a financial situation
E.g., Owing someone $4

4. Use a number line or integer counters to figure out the result of each calculation.

a) −4 + 6
E.g.,

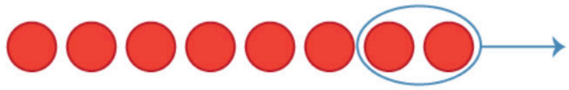


The sum is +2, since there are 2 yellow counters left after I group red and yellow counters to make 0.

Integers (continued)

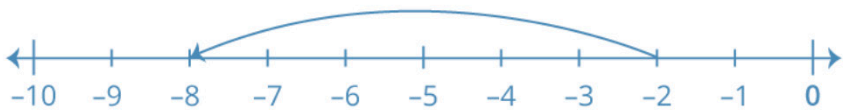
b) −8 − (−2)

E.g.,



The difference is −6. There are 6 red counters left after I remove 2.

OR



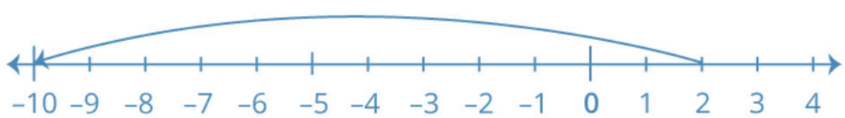
The answer is −6, since I would have to go 6 to the left from −2 to get to −8.

c) −10 − 2

E.g.,

The answer is −12, since that's where I'd land when I go 2 to the left from −10.

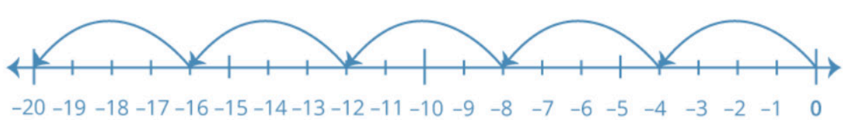
OR



The answer is −12, since I'd have to go back 12 to get from 2 to −10.

d) 5 × −4

E.g.,



The product is −20. I'd take 5 jumps of −4 starting at 0 and end up at −20.

Integers (continued)

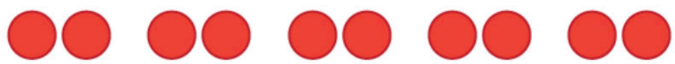
e) $-12 \div -2$

E.g.,

The answer is 6, since I can make 6 groups of −2 out of −12.

f) $-10 \div 5$

E.g.,



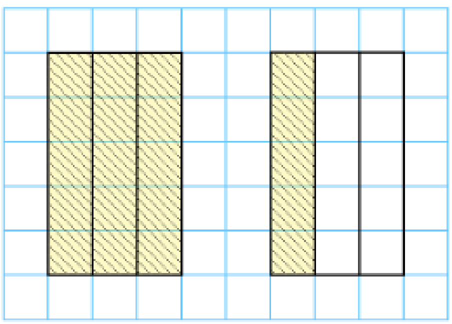
The answer is −2. If you make 5 equal groups out of 10 red counters, there are 2 red counters, which is −2, in each group.

Multiplying and Dividing Fractions

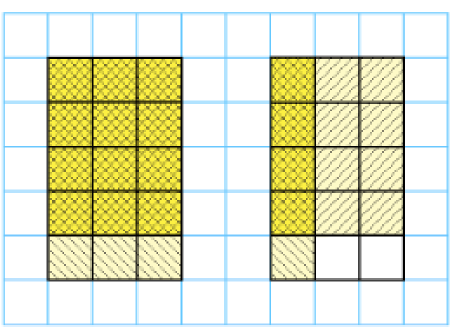
1. Draw a model to show how to determine each answer. Explain your reasoning.

a) $\frac{4}{5} \times \frac{4}{3}$

E.g., $\frac{4}{5} \times \frac{4}{3}$ means $\frac{4}{5}$ of $\frac{4}{3}$, so I started with $\frac{4}{3}$. I drew rectangles, divided them into thirds, and shaded four of the thirds.



To get $\frac{4}{5}$ of what I shaded, I divided the rectangles into fifths by going across. Then I shaded $\frac{4}{5}$ of that.



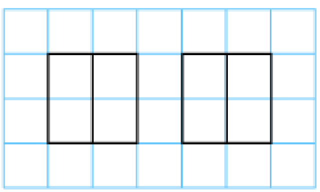
The answer is the overlap part. I can see it's 16 little sections, and each section is $\frac{1}{15}$. So, $\frac{4}{5} \times \frac{4}{3}$ is $\frac{16}{15}$.

Multiplying and Dividing Fractions (continued)

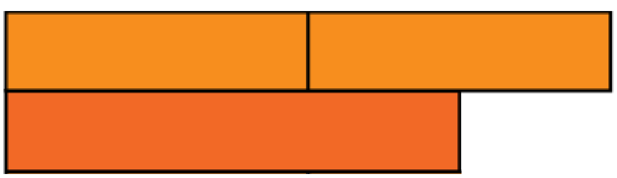
b) $\frac{8}{3} \div \frac{1}{2}$

E.g., This really asks how many halves fit into $\frac{8}{3}$.
But $\frac{8}{3}$ is the same as $2\frac{2}{3}$.
I decided to work with the whole number and fraction parts separately.

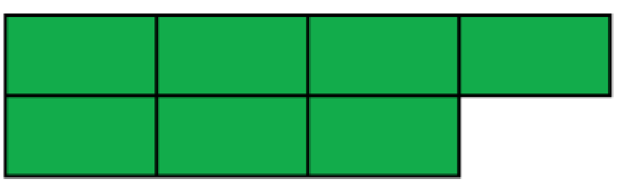
2 wholes divided into halves is 4. I know that because there are 2 halves in each whole.



For the $\frac{2}{3}$ part, I decided to use fraction strips. I want to see how many halves fit into $\frac{2}{3}$.



There's one $\frac{1}{2}$ strip and a bit more.
To get the exact amount, I can replace the pieces with sixths.



Four $\frac{1}{6}$ pieces cover the $\frac{2}{3}$. Three $\frac{1}{6}$ pieces cover the $\frac{1}{2}$.
I need one more $\frac{1}{6}$ piece to even them out.
But another $\frac{1}{6}$ is $\frac{1}{3}$ of another strip of $\frac{3}{6}$,
which means that $\frac{1}{2}$ fits $1\frac{1}{3}$ times.

So, $\frac{8}{3} \div \frac{1}{2} = 4 + \frac{4}{3}$, or $5\frac{1}{3}$.

Multiplying and Dividing Fractions (continued)

2. Calculate.

a) $4\frac{1}{2} \times 3\frac{2}{3}$

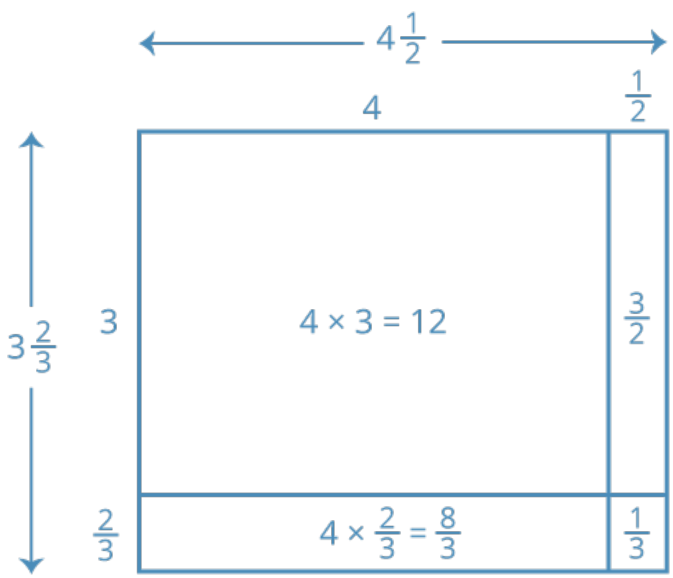
$16\frac{1}{2}$

E.g., I thought about it as $\frac{9}{2} \times \frac{11}{3}$, which is $\frac{99}{6}$, or $16\frac{1}{2}$.

That's because $\frac{9}{2} \times \frac{11}{3} = \frac{9 \times 11}{2 \times 3}$.

OR

I added in parts. $4\frac{1}{2} \times 3\frac{2}{3}$ is the same as $4 \times 3 + \frac{1}{2} \times 3 + 4 \times \frac{2}{3} + \frac{1}{2} \times \frac{2}{3}$.



$12 + 1\frac{1}{2} + 2\frac{2}{3} + \frac{1}{3}$

$13\frac{1}{2} + 3 = 16\frac{1}{2}$

b) $\frac{20}{3} \div \frac{3}{2}$

$\frac{40}{9}$ or $4\frac{4}{9}$

E.g., I wrote this as $\frac{40}{6} \div \frac{9}{6}$, which means I have the same units for both parts, so this is $\frac{40}{9}$.

OR

I thought of this as $\frac{20}{3} \times \frac{2}{3} = \frac{40}{9}$.

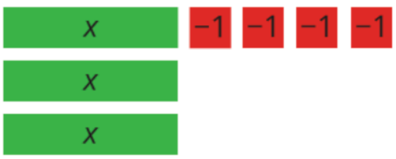
Name: ______________________ Date: ______________

Algebraic Thinking

1. Use algebra tiles or sketches to represent each algebraic expression.

a) $3x - 4$

E.g.,



b) $5 - 3x$

E.g.,

2. Why can the blue section in the diagram be described as $30 - n$?

E.g., It's how much longer 30 is than n, and that's what $30 - n$ means.

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Algebraic Thinking (continued)

3. If you know the value of n, how would you determine the value of each expression?

a) $4n + 8$

E.g., I would multiply the value by 4 and then add 8.

b) $20 - 3n$

E.g., I would triple the value and then subtract that number from 20.

c) $8n - \frac{n}{3}$

E.g., I would multiply the value by 8 and also divide it by 3 and then find the difference.

4. Without solving the equation below, tell what you know about n.

$3n - \frac{n}{2} = 15$

E.g., I know that n is less than 15.

OR

$2\frac{1}{2}$ of n is 15, so n is less than 7.

5. Solve each equation. Check your solutions.

a) $2x - 3.5 = -21.5$

E.g., $2x = -18$

$x = -9$

The solution is correct because both sides are −21.5 when $x = -9$.

b) $5x - 8 = -2x + 13$

E.g., $7x = 21$

$x = 3$

The solution is correct because both sides are 7 when $x = 3$.

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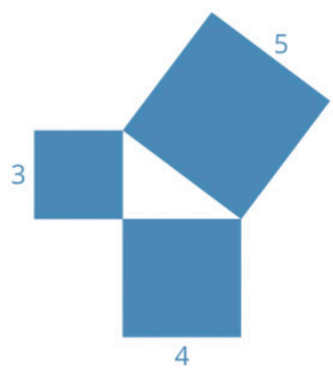
Pythagorean Theorem

1. A designer wants to create a logo based on a right triangle. One sketch includes a triangle with side lengths of 36 units, 20 units, and 25 units. Is it a right triangle? How do you know?

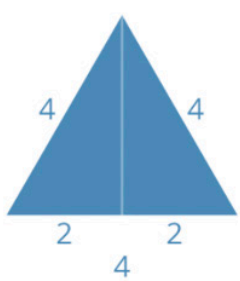
No.
E.g., A right triangle would have this side length relationship: $36^2 = 20^2 + 25^2$.

$36^2 = 1296$
$20^2 + 25^2 = 400 + 625 = 1025$, which is less than 1296, so this is not a right triangle.

2. How could you use a diagram to show that 9-12-15 describes the side lengths of a right triangle?
E.g., I would cut out a 9-by-9 square, a 12-by-12 square, and a 15-by-15 square and show that if I join them at the corners, they form a right triangle between them.



3. An equilateral triangle has a side length of 4 cm. What is its area? Show your thinking.
7 cm^2. E.g., I know the base could be split into two lengths of 2 cm, so the height would be the height of a right triangle with a hypotenuse 4 cm long and leg 2 cm long. That means the height would be 3.5 cm, so the area would be $\frac{1}{2} \times 3.5 \text{ cm} \times 4 \text{ cm}^2 = 7 \text{ cm}^2$.



MATH UP Name: ______________________ Date: ________________

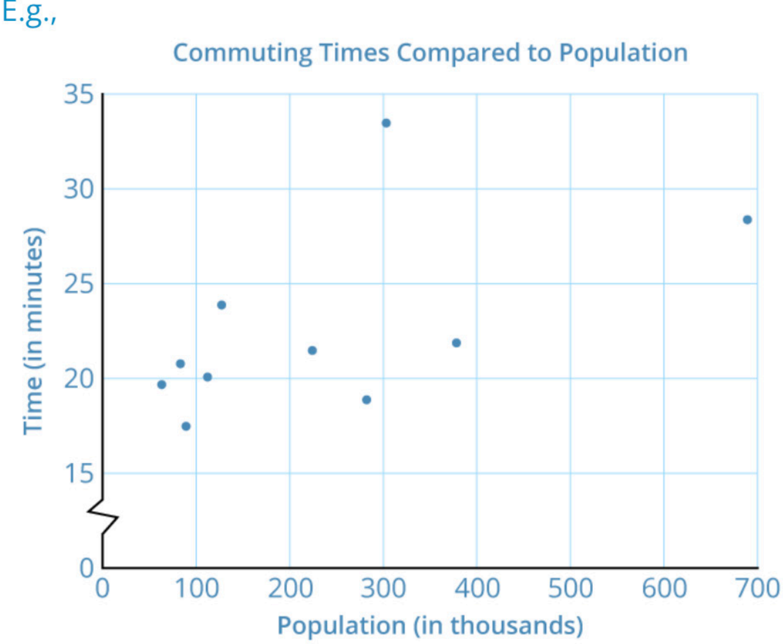
Scatter Plots

1. Create a scatter plot to show each set of data.

a) average commuting times in 10 urban areas

Urban Area	Population (in thousands)	Average commute time (in minutes)
Belleville	68	19.7
Guelph	132	23.9
Hamilton	694	28.4
Kingston	117	20.1
London	383	21.9
Oshawa	308	33.5
St. Catharines – Niagara Falls	229	21.5
Sudbury	88	20.8
Thunder Bay	94	17.5
Windsor	287	18.9

E.g.,



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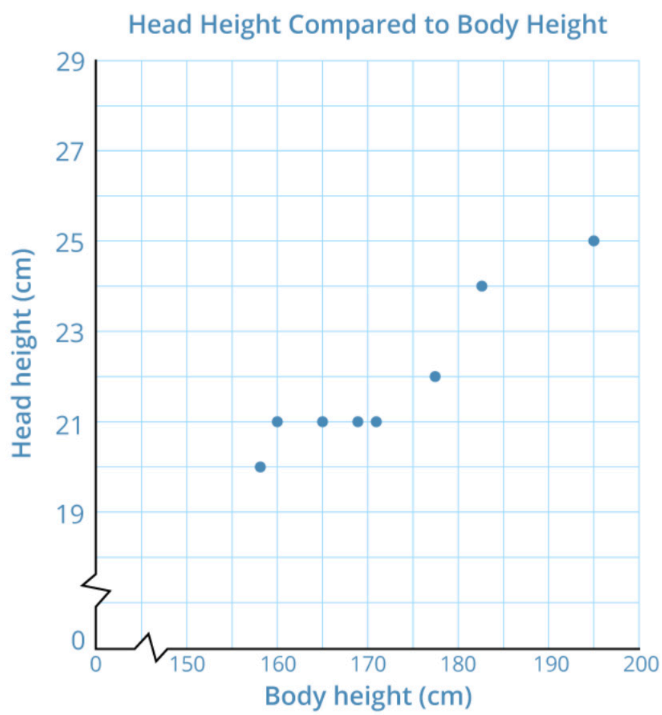
Scatter Plots (continued)

b) height of head compared to height of body for eight people

Body height (cm)	Head height (cm)
160	21
171	21
183	24
195	25
177	22
165	21
158	20
169	21

E.g.,



Scatter Plots (continued)

2. How do you choose scales for your axes when creating a scatter plot?
E.g., I think of the range for each variable and mark numbers on the axis that allow the whole range of numbers to be shown without the graph being too big.