Grade 9 Summer Learning

For students who have just completed Grade 9

Much of Grade 9 is learning about dealing with relations, mostly linear relations. More generally, students learn to represent relations in different ways, describe their characteristics, solve equations involving that sort of relation, and transform them geometrically. Because students will do similar things with quadratic relations in Grade 10, ensuring that students understand these ideas with simpler linear relations is critical.

As well, three other topics are important to revisit. The topic of powers is revisited since it will be important as exponential functions are introduced. Ratios, Rates, and Percent are included since they are so fundamental in trigonometry. The notion of solving problems with right triangles is introduced since so much of trigonometry is built on work with right triangles. Therefore, topics that might be the focus of a Grade 9 summer program to ready students for Grade 10 involve the following:

- Powers
- Ratios, Rates, and Percent
- Linear Relations
- Solving Equations
- Characteristics of Relations
- Relating Two Variables
- Geometric Properties

Essential Understandings that are the focus of the support:

RN–1 Every real number can be represented in many ways. Each way highlights something different about that real number.

O–3 Multiplication is about a change from a unit of a given size to a unit of 1. You know the size and the number of units (the size and the number of groups), and you multiply to figure out the number of units of 1 (the product).

O–4 Division is about a change from a unit of 1 to a unit of a given size. You know the number of units of 1 and the size of the unit (the dividend and the size of the group), and you divide to figure out the number of units (the number of groups). Or you know the number of units of 1 and the number of units (the dividend and the number of groups), and you divide to figure out the size of the unit (the size of the group).

O–7 Performing operations with numbers is often made easier by decomposing and recomposing numbers and/or by thinking of numbers in other units.

PR–1 Sometimes it is useful to compare two numbers in terms of how far apart they are, but other times it is useful to compare them in terms of how many units of one number it would take to fit into the other.

PR–4 Percent can be used to standardize the comparison of, or relationship between, two numbers.

PR–5 Any rate or ratio relationship can be represented in different ways. Different representations might be useful in different situations.

FR–1 A function or relation can be represented in a variety of ways. Each representation might highlight something different about the function or relation.

FR–3 Classifying a function or relation provides lots of information about it.

FR–4 The transformations that are fundamental to determining the relationships between functions or relations of a certain type have predictable effects on the algebraic descriptions of those relationships.

A–2 Equality is an expression of balance. The two sides of an equation describe the same quantity.

A–4 Often more than one equation or one set of related equations can describe a situation algebraically.

A–6 Solving an equation uses relationships between numbers and relationships between operations to determine an equivalent, simpler form of the equation.

A–7 Knowing a relationship between two variables allows you to predict information about one based on what you know about the other.

A–8 Any algebraic situation can be represented in many ways.

DA–3 Often a visual data display makes it easier to show data. The type of graph used depends on what we want viewers to see, including frequency (how often something occurs), comparisons between categories, changes over time, and so on.

DA–4 Interpreting data involves not only reading information but also drawing conclusions, and sometimes it involves using those conclusions to make predictions and inferences.

DA–5 Sometimes it is useful to represent a set of data in a single meaningful number to quickly summarize it; there are choices in what that number can be.

M–6 Some measurements of an item are independent of other measurements of that item, but some are related.

M–7 Sometimes known measurements can be used to calculate unknown measurements.

SO–5 Geometric constructions are based on properties of various geometric shapes.

Grade 9 Summer Learning

For students who have just completed Grade 9

I have set this up for 20 sessions of about 1.5–2 hours each.

- Each day includes at least one number talk.
- Each day also includes either a diagnostic task, perhaps followed up by an additional number talk or practice activities, or a MathUP lesson to be followed up with practice activities.

Number Talks

Number talks that are particularly recommended are the following:

Grade 9: 2, 4, 6, 16, 17, 19, 20, 21, 22, 23, 28, 29, 30, 32, 36, 40, 42, 44, 45

Diagnostic Tasks

If MathUP Grade 10 existed, I would be using the diagnostic tasks from Grade 10 to help you discover what students had learned from Grade 9.

But MathUP Grade 10 does not exist. So, what I decided to do was to use some of the questions that I considered the most critical parts of what students should have learned from Grade 9 topics so that you can see what they have learned and what they have not.

On a day that a diagnostic task is used (based on the seven focus topics), there would be a number talk followed by the diagnostic task. The task should not be described as a test, but just as an activity to reduce anxiety.

A PDF of the diagnostic tasks is provided.

It might be appropriate to review some of the vocabulary in the diagnostic task before administering it.

Should students struggle with these tasks, you might even go back to the related Grade 9 diagnostic tasks and treat them as additional activities.

If there are no problems with the diagnostic task and you have more time to work with students, you might choose to work on additional number talks, or you might choose to use one or more of these additional activities:

Minds On Activities from the following:

- Number Systems and Properties, Lesson 2
- Algebraic Thinking, Lesson 1
- Algebraic Thinking, Lesson 2
- Algebraic Thinking, Lesson 3
- Characteristics of Relations, Lesson 1

Cross-Strand Tasks: Powerful Relationships, And the Limit Is
Games from these topics: Powers, Integers, Algebraic Thinking, Linear Relations, Solving Equations, Characteristics of Relations

Assuming that you work with students at the Grade 9 level and do not see the need to go back farther, the suggested MathUP lessons to focus on are listed below.

Before you deliver a lesson, it would be valuable for you to first read the Sum It UP page to see what content is being covered.

Then you can use the three parts of the lesson, Minds On, Action, and Consolidate, followed by the Your Turn and additional practice activities.

MATH UP Grade 9 Summer Learning

For students who have just completed Grade 9

Strand	Topic	Lesson	* Prerequisite Topics
N	Powers	**Lesson 1** Integer Exponents **Lesson 2** Simplifying Expressions Involving Powers	None
N	Ratios, Rates, and Percent	**Lesson 2** Solving Rate Problems **Lesson 3** Solving Percent Problems	None
A	Linear Relations	**Lesson 1** Graphing Linear Relations **Lesson 2** Representing Linear Relations in Many Ways	None
A	Solving Equations *	**Lesson 1** Solving Equations Algebraically	Algebraic Thinking
A	Characteristics of Relations *	**Lesson 2** Graphing Simple Relations and Inequalities **Lesson 3** Transformations of Lines **Lesson 4** Solving Problems Using Equations of Lines	Linear Relations
D	Relating Two Variables *	**Lesson 1** Describing the Strength of Relationships **Lesson 2** Regression Models	Characteristics of Relations
GM	Geometric Properties	**Lesson 3** Solving Problems Involving Right Triangles	None

MATH UP Name: ______________________ Date: ________________

Powers

1. Evaluate each expression.

a) $3^4 \times (-2)^{-3}$

$-\frac{81}{8}$

E.g., $81 \times \frac{1}{(-2)^3} = 81 \times \left(-\frac{1}{8}\right)$

b) $5^3 \times 4^3$

8000

E.g., $125 \times 64 = 8000$

c) $a^{-3} \times b^6$ when $a = 1$ and $b = 10$

E.g., 106, or 1 000 000

2. Simplify each expression to one power that is equivalent.

a) $4^5 \times 4^{-8}$

E.g., 4^{-3} OR 2^{-6}

b) $2^5 \div 2^6$

E.g., 2^{-1} OR $\left(\frac{1}{2}\right)^1$

c) $2^6 \times 3^6$

E.g., 6^6 OR 36^{-3}

Powers (continued)

3. Write each number as the product of a power with a positive exponent and a power with a negative exponent.

a) 250
E.g., $10^3 \times 2^{-2}$

b) $\frac{25}{64}$

E.g., $5^2 \times 4^{-3}$

c) 6.25
E.g., $10^2 \times 2^{-4}$

d) 7200
E.g., $30^2 \times \left(\frac{1}{2}\right)^{-3}$

4. a) How would you explain why $8^3 \times 8^{-5} = 8^{-2}$?

E.g., $8^{-5} = \frac{1}{8}^{5}$, so $8^3 \times 8^{-5}$ is $\frac{8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8} = \frac{1}{8 \times 8} = 8^{-2}$

b) How would you explain why $9^{12} = 3^{24}$?
E.g., $9^{12} = 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
Each 9 is 3×3, so if there are twelve 9s, that would make twenty-four 3s, which is 3^{24}.

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Ratios, Rates, and Percent

1. Determine an equivalent ratio for 17:28, where the second term is greater than 100.
E.g., 170:280

2. Are these ratios or rates equivalent? Show why or why not.

a) 6 for \$8 and 8 for \$9
No. E.g., If 6 costs \$8, then each costs a bit more than \$1, so if you add 2 items, the increase would be more than \$2 extra, and it's not.

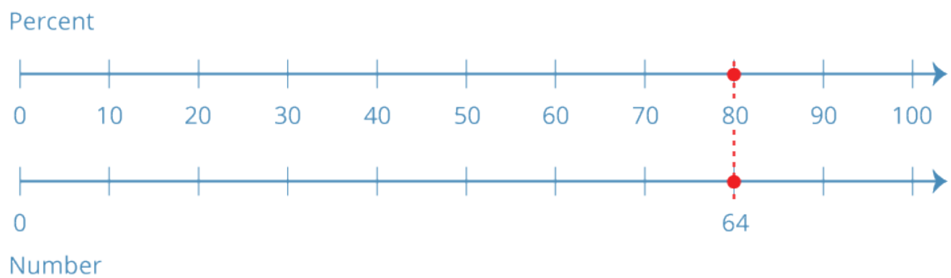
b) 8.2 L/100 km and 4.2 L/50 km
No. E.g., They are different since if you halve the kilometres, you would halve the 8.2 to get 4.1.

3. Evaluate: 124% of 50
62

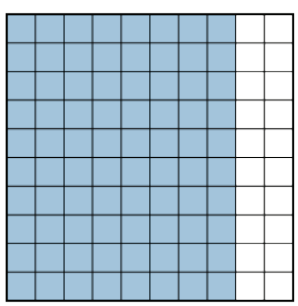
4. What is the whole if 75% is 54?
72
E.g., Since $\frac{3}{4}$ is 54, then $\frac{1}{4}$ is 18, and so the whole is $4 \times 18 = 72$.

Ratios, Rates, and Percent (continued)

5. Draw a diagram that would explain why it makes sense that 80% of 80 is 64.
E.g.,



Since you want 80% of 80, the number 80 goes below 100%. That means that under each 10% mark should be the corresponding multiples of 8 to get to 80 after 100%. That means that under 80% would be 8 × 8 = 64.



I can see that each column is worth 8 for the whole grid to be worth 80, and so 8 columns are worth 64.

6. An item is sold for $100 after markup.
What might the store have paid to get the item?
Give two possible prices with the associated markups.
E.g., There are a lot of possible answers.
It would be a 100% markup if the original price were $50, since you'd be adding $50 to itself.
It would be a 33.33% markup if the original price were $75, since you'd be adding $\frac{1}{3}$ of that, which is $25.

Ratios, Rates, and Percent (continued)

7. A recipe for chicken curry uses 300 mL coconut milk, 375 mL chicken stock, and 200 mL frozen peas. A cook has only 200 mL coconut milk. How should the cook adjust the other measurements?

E.g., I think that since the ratio is 2:3, I would take $\frac{2}{3}$ of the other amounts. So, I would use 250 mL chicken stock and 140 mL of frozen peas.

8. You raise a price by 25%, and then later you give a 25% discount on the higher price. Are you back to the original price? Explain.
E.g., No. You added 25% to a smaller number than the number you took 25% away from.

9. How do you know that there is no exact solution to this problem?
In a room, the ratio of Grade 9 to Grade 10 students is 4:5.
There were 80 students altogether. How many were Grade 9 students?
E.g., If the ratio is 4:5, the total number of people has to be a multiple of 9 (4 + 5), and 80 is not.

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Linear Relations

1. What is the rate of change of *y* for a change of 1 in *x* for each linear relationship?

a) $y = 50 - 4x$

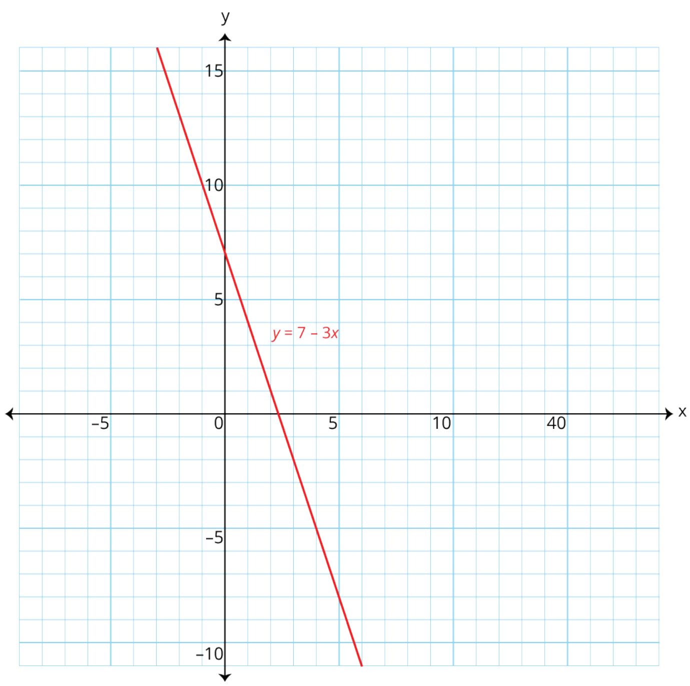
–4

b)

x	*y*
0	3
1	5.5
2	8
3	10.5
4	13

2.5

c)



–3

Linear Relations (continued)

2. a) Describe the visual below using an algebraic relationship.

y						
x	*x*	*x*	*x*	*x*	*x*	12

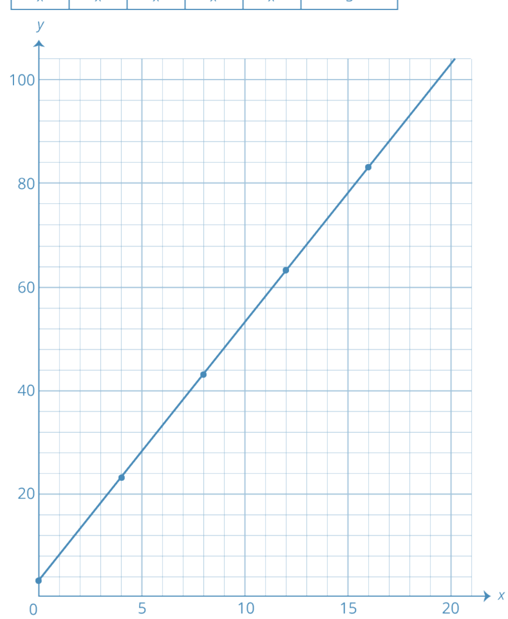
$y = 6x + 12$

b) Describe the visual below using an algebraic relationship.

x	*y*
0	3
1	8
2	13
3	18
4	23

E.g.,

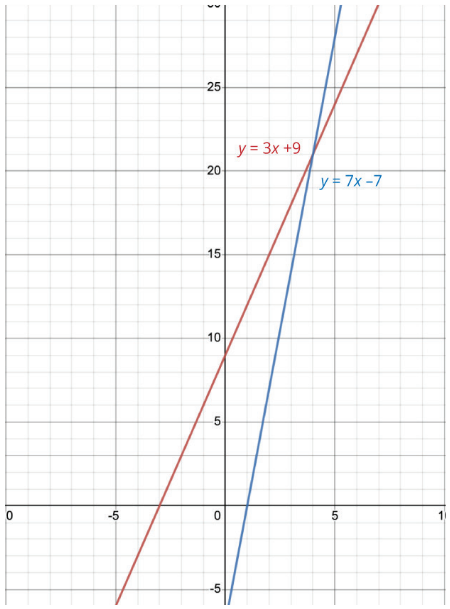
y					
x	*x*	*x*	*x*	*x*	3



Linear Relations (continued)

3. Use a graph to determine where y = 3x + 9 and y = 7x − 7 intersect.

E.g.



They intersect at (4, 21).

4. Use an equation to show where y = 3x + 9 and y = 7x − 7 intersect.

E.g., 3x + 9 = 7x − 7

That is the same as 4x = 16, so x = 4.

3 × 4 + 9 = 21, so the relations intersect at (4, 21).

5. a) Why is a graph useful for deciding whether a relationship is linear?

E.g., On a graph, you can just see whether the relationship forms a line.

b) When might an equation be more useful than a graph for describing a linear relationship?

E.g., A graph might not be big enough to show the value of the variables when one or both of them are very large. An equation would be more useful then.

Linear Relations (continued)

6. One electrician charges $50 to come for a repair and $90 per hour to do the work. A different electrician charges $100 to come for a repair and $75 per hour to do the work.

a) How do you know that both situations describe linear relations?
E.g., They both have a constant rate of change — the rate per hour.

b) Why would the graphs of the two relations intersect?
E.g., They grow at different rates. Even though the first electrician charges less to start with, their hourly fee is greater, so eventually the cost for that electrician will catch up to and even pass the cost for the other electrician.

c) What information does that intersection point tell you?
E.g., It tells you for how many hours of work the fees for the electricians would be the same.

d) Why does solving $50 + 90h = 100 + 75h$ tell you the first coordinate of the intersection point?
E.g., The h values for both linear relations are equal at that point.

e) Why can you substitute the h value from part d) into either linear relation to figure out the second coordinate of the intersection point?
E.g., The whole idea of an intersection point is that the first and second coordinates are the same for both linear relations at that point.

MATH UP Name: ______________________ Date: ______________

Solving Equations

1. What equation or equations does this model represent?

x	*x*	*x*	4.8
x	8.7		

E.g., $3x + 4.8 = x + 8.7$

2. Is −30 a solution to the equation below? Explain.

$7x - 19 = 42 + 5x$

No. E.g., I estimated that the left side would be about −200 and the right side would be closer to −100 if x were to equal −30.

3. Solve each equation.

a) $-3x + 8.75 = 2x + 0.8$

E.g.,

$-3x + 8.75 = 2x + 0.8$

$-5x + 8.75 = 0.8$

$-5x = -7.95$

$x = 1.59$

b) $4x - 3 = -x + 8$

E.g.,

$4x - 3 = -x + 8$

$5x = 11$

$x = 2.2$

Solving Equations (continued)

3. Solve each equation. (continued)

c) $-5x - 1 = -3x + 9$

E.g.,

$-5x - 1 = -3x + 9$

$-2x = 10$

$x = -5$

4. Estimate the solution to the following equation, and explain your thinking.

$-4x - 20 = 3x - 81$

E.g., Since $7x$ is about 60, x is about 8.

5. Draw a diagram for the following equation, and describe how it might help you estimate the solution.

$5x - 8 = 2x + 16$

E.g.,

<table>
<tr><td>x</td><td>x</td><td>x</td><td>x</td><td>x</td><td>−8</td></tr>
<tr><td>x</td><td>x</td><td colspan="4">16</td></tr>
</table>

I can see that $3x - 8$ balances 16, so x must be about 8.

6. Create an equation you could solve to help you figure out what you need to determine in the following situation:

I want to know how much 1 kg of nuts costs if 4.2 kg of nuts and a package of pasta for $2.29 costs the same as 2 kg of nuts and some packages of cheese that cost $17.55.

E.g., $4.2k + 2.29 = 2k + 17.55$

Solving Equations (continued)

7. How do you know the solutions to these two equations are the same?
$-8x - 25 = x - 7$ and $18x = -36$
E.g., The equations are equivalent since you kept the balance by adding $8x$ and 7 to both sides and multiplying both sides by 2.

8. How do you know that the solution to $36x + 60 = 120x + 14$ cannot be an integer even before you solve it?
E.g., It cannot be an integer. The left side is a multiple of 3 + a multiple of 3, which is a multiple of 3. But the right side is a multiple of 3 + a non-multiple of 3, which is not a multiple of 3.

9. How do you know that if $5x + 17 = 3x + 2$, x must be negative?
E.g., If x were positive, then $5x$ would be more than $3x$, and 17 is more than 2. So, the left side would be more than the right side not equal to it.

Name: ______________________ Date: ______________

Characteristics of Relations

1. Which of these tables of values describe linear relations?

Table 1

x	y
1	4
2	2
3	0
4	–2

Table 2

x	y
1	8
2	10
3	12
4	14

Table 3

x	y
0	–10
1	–20
2	–15
3	10

Table 4

x	y
0	6
–2	4
–4	6
–8	10

Tables 1 and 2

2. Which of these equations describe linear relations?

A: $2x + 3y = 4$

B: $2x^2 - y = 5$

C: $3x^3 + y^2 = 5$

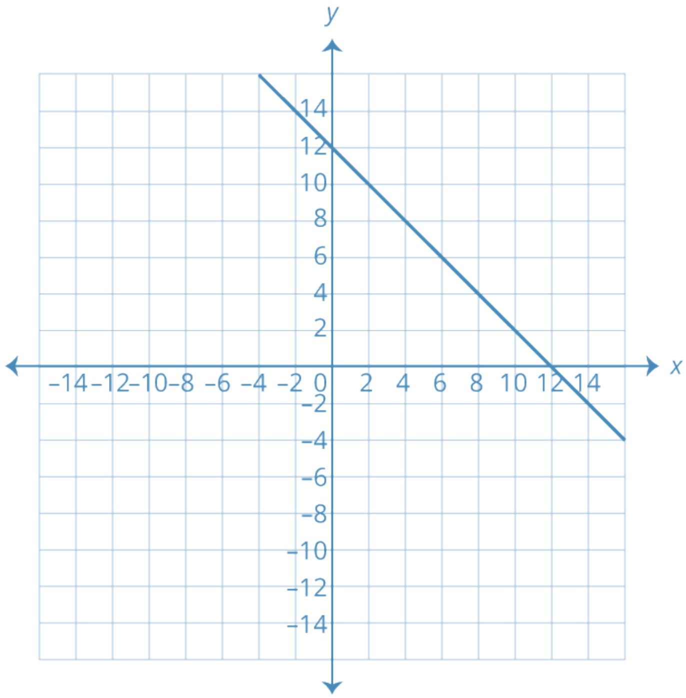
D: $3x = 2y - 2$

A and D

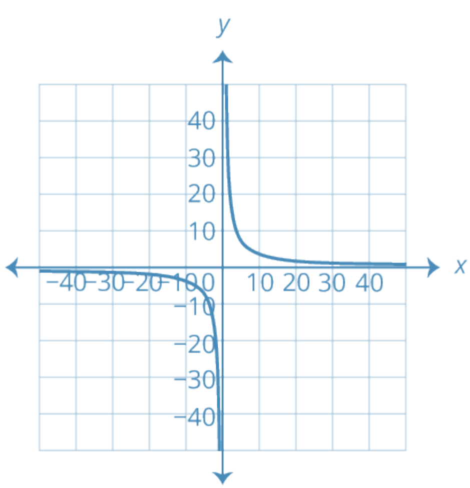
Characteristics of Relations (continued)

3. Graph each equation or inequality.

a) $x + y = 12$



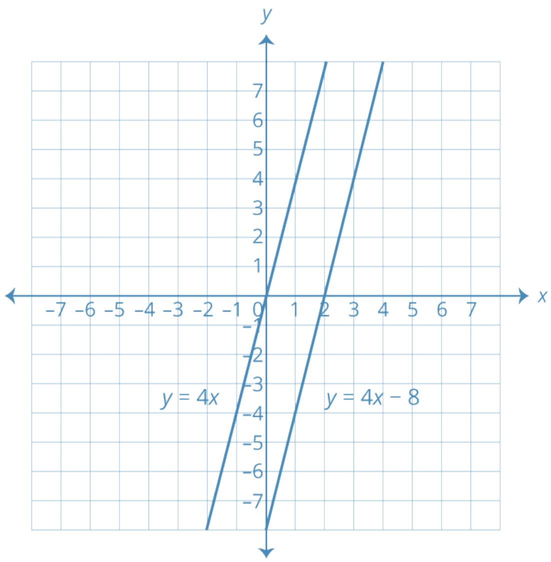
b) $xy = 36$



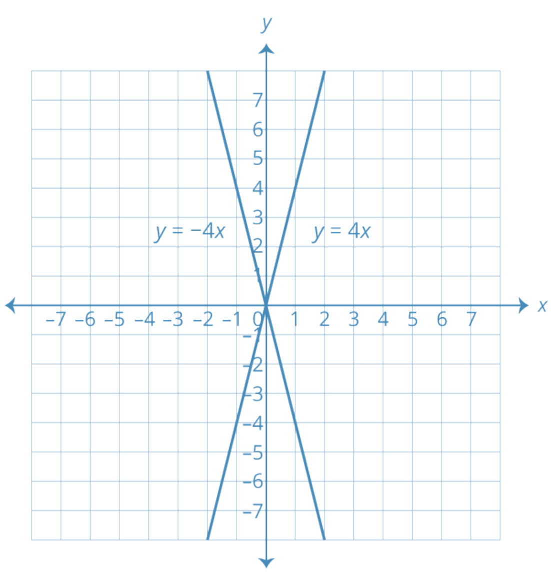
Characteristics of Relations (continued)

4. Perform the following transformations of $y = 4x$. Then write an equation for the new line.

a) Translate right 2 units



b) Reflect in the y-axis



Characteristics of Relations (continued)

5. How can you figure out the slope of a line?

E.g., I pick two points on the line. I measure how many units to the right or left and then how many units up or down I have to go to get from one to the other. I divide the rise by the run.

6. Determine the equation of the line that goes through (4, 8) and (6, 12). Explain your thinking.

$y = 2x$

E.g., I can see that both times, the y is double the x.

7. Include the missing values in this table of values to make this relationship linear.

x	***y***
0	
1	12
2	16
3	

E.g.,

x	***y***
0	8
1	12
2	16
3	20

Characteristics of Relations (continued)

8. Predict, without graphing, what will be the same and what will be different about these two lines: $x - y = 12$ and $x - y = 14$. Explain your thinking.
E.g., They have the same slope; it's 1 since when x increases by 1, so does y for both of them. But the first line has intercepts at (0, −12) and (12, 0) and the second at (0, −14) and (14, 0).

9. How can you predict that the y-intercept will change whether you translate the line $y = 2x$ to the right or down?
E.g., If I do a translation, I keep the slope the same. There is only one line with a slope of 2 with the y-intercept of 0, so if the line is parallel, the y-intercept has to change.

10. How do you know the lines $y = 2x$, $y = 2x - 4$, and $y = 2x + 6$ are parallel?
E.g., For each line, when x gets 1 bigger, y gets 2 bigger since $2(x + 1) = 2x + 2$. So, the rate of change, which is the slope, doesn't change.

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Relating Two Variables

1. Use words such as weak, strong, positive, or negative to describe the value of the following correlations between two variables.

a) −1
E.g., a very strong, negative relationship

b) 0
E.g., no relationship at all

c) 0.8
E.g., a strong, positive relationship

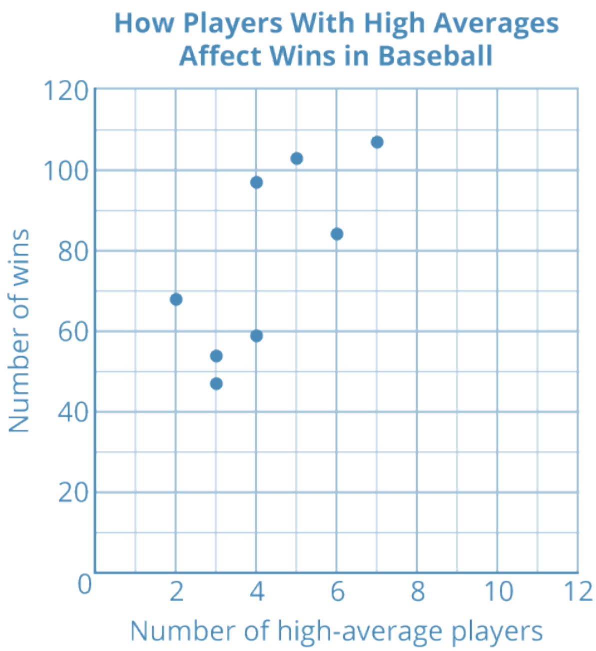
2. A sports journalist collected data relating the number of baseball players on a team with high batting averages and the number of wins for that team in a regular season. Display the data with a scatter plot.

Number of high-average players	Number of wins
5	103
6	84
3	54
4	59
3	47
4	97
7	107
2	68

Relating Two Variables (continued)

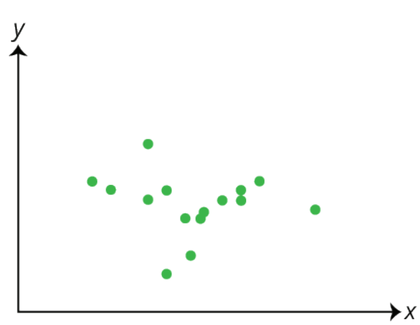
2. A sports journalist collected data relating the number of baseball players on a team with high batting averages and the number of wins for that team in a regular season. Display the data with a scatter plot. (continued)

E.g.,



3. Choose a correlation from the numbers below that best fits each set of data.
–0.7 –0.1 0.3 0.9

a)

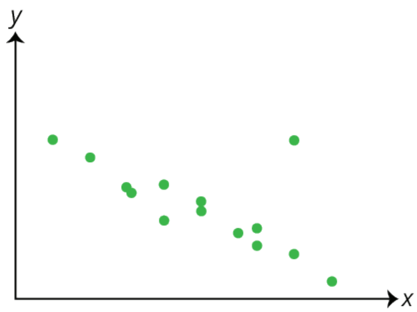


0.1

Relating Two Variables (continued)

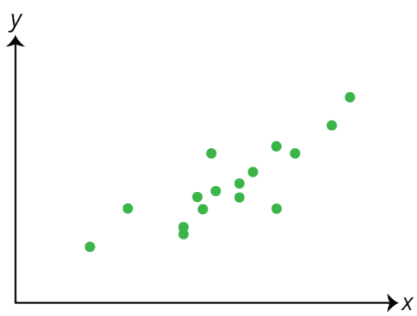
3. Choose a correlation from the numbers below that best fits each set of data.
–0.7 –0.1 0.3 0.9 (continued)

b)



–0.7

c)



0.8

4. Predict the value of the correlation between each set of variables.
Explain your thinking.

a) Celsius and Fahrenheit temperatures
E.g., The correlation is 1 because there is a linear relationship between the two values. If you know one, you can calculate the other. Also, as one increases, so does the other.

Relating Two Variables (continued)

4. Predict the value of the correlation between each set of variables. Explain your thinking. (continued)

b) the area of a rectangle and one of its side lengths
E.g., The correlation would be 0 since just knowing one side length doesn't give you any information about the area. You need to know more to figure out the area.

c) the amount of snow on the ground and how long it takes to clear a sidewalk
E.g., I predict a correlation that is pretty high since if there is more snow, it would probably take longer, but I don't think it'll be perfect, because there are a lot of different ways to clear snow from sidewalks, and also some snow is heavier than other snow.

5. What is the purpose of calculating a regression model for a set of data?
E.g., Regression models are a way of figuring out possible values of one variable when you know the value of the other variable. They are tools used for estimating and predicting.

6. Explain why the line $y = 5x + 10$ would likely be a poor regression model for the data below.

x	1	2	3	4	5	6	7	8	9	10	11
y	15	19	27	26	28	32	33	40	40	35	44

E.g., I used the model for a few different x values from the table, and the results were not close to the y values in the table.

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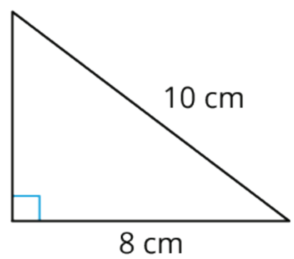
Geometric Properties

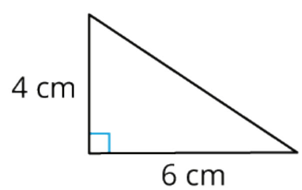
1. What is the unknown length in each case?

a)



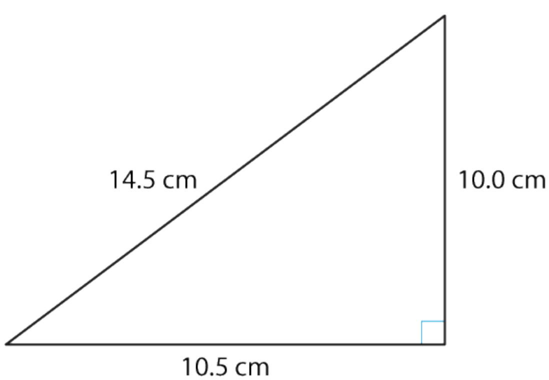
6 cm

b)



$\sqrt{52}$ cm, or about 7 cm

2. In this right triangle, two side lengths are close to the same length.



Name: ______________________________ Date: ____________________

Geometric Properties (continued)

2. In this right triangle, two side lengths are close to the same length. (continued)

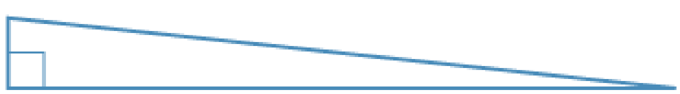
a) Are any of the angles also close in size?

E.g., Yes, the two angles that are not right angles are close in size. They are both about 45°, but not exactly.

I figured this out by noticing that this triangle is very close to an isosceles triangle; that is, two sides are very close to equal. Isosceles triangles also have two equal angles, so the two angles in this triangle must be also close to equal. Since the other angle is a right angle, these two angles would be close to half of 180° – 90°, or close to 45° each.

b) Could there be a different right triangle that has two side lengths that are close but where one is the hypotenuse?

E.g., Yes. I drew this example.



3. A right triangle has two equal side lengths. Decide what they are, and figure out the length of the third side.

E.g., 1 and 1 and $\sqrt{2}$

4. Describe two different right triangles with a hypotenuse of 12 that have different areas. Tell how you know the areas are different.

E.g., One triangle could be 2 – $\sqrt{140 – 12}$. The area is $\sqrt{140}$, which is about 12.

Another could be 10 – $\sqrt{44 – 12}$. The area is $5\sqrt{44}$, which is about 33.

